But the integrand of the first of these two integrals is less than (or equal to) $k^n\sqrt{1+n^2k^{n-2}}$ and since the limit of this is 0, the limit of the integral is zero also. To handle the second integral, write the integrand as

$$nx^{2n-1}\sqrt{1+\frac{1}{n^2x^{2n-2}}}.$$

We may now develop in series and show that the limit of every term after the first is zero, or we may proceed as follows:

$$\int_k^1 nx^{2n-1}\sqrt{1+\frac{1}{n^2x^{2n-2}}}\,dx = \int_k^1 nx^{2n-1}\left\{1+\sqrt{1+\frac{1}{n^2x^{2n-2}}}-1\right\}dx$$

$$= \int_k^1 nx^{2n-1}\cdot dx + \int_k^1 n\cdot x^{2n-1}\cdot\frac{\frac{1}{n^2x^{2n-2}}}{\sqrt{1+\frac{1}{n^2x^{2n-2}}}+1}\cdot dx,$$

by rationalizing the numerator. The limit of the first integral is $\frac{1}{2}$ and since the integrand in the second may be made as small as we please, the limit of the second integral is zero.

Also solved by J. A. Caparo and the Proposer.

379. Proposed by C. N. SCHMALL, New York City.

Express the equation of the folium, $x^3+y^3=3axy$, in parametric form and find the area of the loop.

(From E. B. Wilson's *Advanced Calculus*, p. 296, ex. 5.)

Solution by E. B. Wilson, Mass. Institute of Technology.

Let $y=mx$, then

$$x=\frac{3am}{1+m^3},\qquad y=\frac{3am^2}{1+m^3};$$

the loop being described by values of m from 0 to ∞. By the formulas for area as a curvilinear integral

$$A=-\int_{m=0}^{\infty}ydx=-\int_0^{\infty}9a^2\frac{m^2(1-2m^3)dm}{(1+m^3)^3}=-\int_0^{\infty}3a^2\frac{1-2u}{(1+u)^3}du,$$

where $u=m^3$. Then

$$A=-3a^2\left[\frac{2}{1+u}-\frac{3}{2}\frac{1}{(1+u)^2}\right]_0^{\infty}=\frac{3}{2}a^2.$$

Also solved by Elijah Swift, C. E. Horne, Wilson L. Miser, W. C. Eells, Horace Olson, J. A. Caparo, H. L. Agard, L. G. Weld, and the Proposer.

MECHANICS.

297. Proposed by C. N. SCHMALL, New York City.

A shrapnel shell strikes the ground and then explodes, dispersing its fragments in all directions with a common velocity v. If a be the area of the ground covered by the fragments, and if the dimensions of the shell be neglected, show that $a=\pi v^4/g^2$.

Solution by Horace Olson, Chicago, Illinois.

According to the laws of physics, the range of a projectile on the horizontal plane from which it is thrown is $(2v^2\sin\theta\cos\theta)/g$ or $(v^2\sin 2\theta)/g$, θ being the inclination with the horizontal of the line of projection. This range has a maximum value, v^2/g, when θ is 45°. Therefore the area of the ground covered by the fragments is $\pi v^4/g^2$, the area of a circle of radius v^2/g.

Also solved by A. M. Harding, J. L. Riley, and P. Peñalver.